

- LIRON COHEN, *Cycles for the sake of induction*.  
Computer Science Department, Cornell University, Ithaca, NY, USA.  
*E-mail:* [lironcohen@cornell.edu](mailto:lironcohen@cornell.edu).

A core technique in mathematical reasoning is that of induction. Formal systems for mathematical reasoning usually capture the notion of inductive reasoning via one or more inference rules that express the general induction schemes, or principles, that hold for the elements being reasoned over. Increasingly, we are concerned with not only being able to formalize as much mathematical reasoning as possible, but also with doing so in an effective way. For this we employ *Transitive closure* logic, which is obtained by a modest addition to first-order logic that affords enormous expressive power. Most importantly, it provides a uniform way of capturing inductive principles. Thus, particular induction principles do not need to be added to, or embedded within, the logic; instead, all induction schemes are available within a single, unified language.

This expressiveness of the logic renders any finitary proof system for it incomplete for the standard semantics. Nevertheless, we develop an infinitary proof theory for transitive closure logic which is complete for the standard semantics. This system captures implicit induction, and its soundness is underpinned by the principle of *infinite descent*. While a full infinitary proof theory is clearly not effective, such a system can be obtained by restricting consideration to only regular infinite proofs (those representable by finite, possibly cyclic, graphs). The uniformity of the transitive closure operator allows semantically meaningful complete restrictions to be defined using simple syntactic criteria. Consequently, the restriction to regular proofs provides the basis for more focussed proof-search strategies, further enhancing the potential for automation.