

Integrating Induction and Coinduction via Closure Operators and Proof Cycles

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Coinductive reasoning about infinitary data structures has many applications in computer science. Nonetheless developing natural proof systems (especially ones amenable to automation) for reasoning about coinductive data remains a challenge. This paper presents a minimal, generic formal framework that uniformly captures applicable (i.e. finitary) forms of inductive and coinductive reasoning in an intuitive manner. The logic extends transitive closure logic, a general purpose logic for inductive reasoning based on the transitive closure operator, with a dual ‘co-closure’ operator that similarly captures applicable coinductive reasoning in a natural, effective manner. We develop a sound and complete non-well-founded proof system for the extended logic, whose cyclic subsystem provides the basis for an effective system for automated inductive and coinductive reasoning. To demonstrate the adequacy of the framework we show that it captures the canonical coinductive data type: streams.